Re-Thinking the use of the OML Model In Electric-Sail Development

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The Orbit Motion Limited (OML) model commonly forms the basis for calculations made to determine the effect of the long, biased wires of an Electric Sail on solar wind protons and electrons (which determines the thrust generated and the required operating power). A new analysis of the results of previously conducted ground-based experimental studies of spacecraft-space plasma interactions indicate that the expected thrust created by deflected solar wind protons and the current of collected solar wind electrons could be considerably higher than the OML model would suggest. Herein the experimental analysis will be summarized and the assumptions and approximations required to derive the OML equation—and the limitations they impose—will be considered.

Nomenclature

A	=	the area of the body
A_w	=	the area of the wire
L	=	the total length of the wire
P	=	impact parameter
S	=	the ion acoustic Mach number $(m_p v^2/2kT_e)^{½}$,
k	=	Boltzmann's constant
e	=	electronic charge
j_{eo}	=	the electron thermal current in the ambient plasma
m	=	electron mass
n	=	particle (ion or electron) number density
r_w	=	the wire radius
r_s	=	the plasma sheath radius
λ	=	the collision mean-free-path length for any species
λ_D	=	the Debye shielding distance $(\varepsilon_o kT_e/ne^2)^{½}$
ϕ_w	=	the electric potential applied to the wire relative to plasma potential
Φ	=	the electric potential normalized by electron thermal energy $(e\phi/kT_e)$

I. Introduction

THE utility of E-sail technology rests on two fundamental issues involving the interaction of a long highly biased wire with the solar wind plasma: (1) the effectiveness of the high-voltage sheath around the positively biased wire in deflecting solar wind protons, which determines the level of thrust achievable; and (2) the magnitude of the electron current collected from the solar wind by the wire, which determines the required input power.

The simple orbit motion limited (OML) model has typically been used to determine the extent of the sheath field surrounding "bare" electrodynamic tethers and the wires of the E-sail array.[1,2] This seems reasonable for small wires biased to high voltages in a very tenuous plasma. However, as shown below, a number of simplifying assumptions and approximations are required in deriving the OML equation. These place conditions on its range of validity and it would appear that some of these conditions are not met in the solar wind for the E-sail application. Moreover, the predicted electron current collection for a wire biased to several kV is less than that crossing a cylindrical area one Debye length in radius. This seems low for a wire biased to such a high voltage. The OML model will, therefore, be considered in some detail in Sections II through V, where the derivation of the OML equation from the Langmuir-Mott Smith model is summarized along with the necessary assumptions and approximations. The fact that these conditions may not hold in this particular application is considered along with the potential effects of their violation. .

To provide a bench-mark check on the OML results, data previously obtained from laboratory simulations of the interaction that occurs between a satellite and its environmental ionospheric plasma was re-evaluated in the context of the E-sail problem. The interaction of a small ionospheric satellite or probe with the ionospheric plasma,

while different is some aspects such as geometry and scale size, is similar in a number of other aspects. The plasma is unmagnatized and collisionless on the scale of the interaction and the sheath-body radius ratio (~ 1) is such that the sheath electric field has a dominant effect on the plasma flow disturbance. Moreover, the electron temperature and density and the ion flow characteristics (flow direction and energy of deflected ion streams) could be accurately measured. This permitted a measurement of the "effective" sheath thickness and its effect on ion steams for various conditions. Predictions based on the laboratory experiments are considerably different from those derived from the OML model. The laboratory experiments and analyses are discussed in Sections VI and VII.

II. General Description of the LMS Model

In 1926, Irvin Langmuir and H. M. Mott-Smith published a theoretical model for the complex plasma sheath phenomenon in which they identified some very special cases which greatly simplified the sheath and allowed a closed solution to the problem.[3] The most widely used application is for an electrostatic, or "Langmuir," probe in laboratory plasma. Adapting the LMS model to real-life conditions is the subject of numerous papers and dissertations. The Orbit-Motion Limited (OML) model that is widely used today is one of these adaptions that is a convenient means of calculating sheath effects outside of the electron retarding region (shown in Fig. 1). The OML equation for electron current collection by a positively biased body is simply:

$$I \cong A\, j_{eo} \frac{2}{\sqrt{\pi}} (\Phi)^{\frac{1}{2}}$$

where A is the area of the body and Φ is the electric potential on the body with respect to the plasma normalized by the electron thermal energy.

Since the Langmuir probe is a simple biased wire immersed in plasma, it is particularly tempting to use the OML equation in calculating the characteristics of the long, highly biased wires of an Electric Sail in the solar wind plasma. However, in order to arrive at the OML equation, a number of additional simplifying assumptions and approximations (beyond those made by Langmuir—Mott-Smith) are necessary. The OML equation is a good approximation when all conditions are met, but it would appear that the Electric Sail problem lies outside of the limits of applicability.

A plasma is defined as an assemblage of neutral and charged particles (positive and negative) characterized by (1) quasi-neutrality; i.e., number of positive and negative charges is, macroscopically, neutral; (2) bulk behavior that is determined by electrostatic and magnetic forces rather than mechanical particle-particle collisions; and (3) large dimensions of the plasma wrt the mean distances between collisions for all constituents (ion-ion, electron-electron and charge-neutral). In the vicinity of a boundary these conditions may break down. For example, an electrically biased surface can form a boundary layer as one species is preferentially absorbed at the surface and quasi-neutrality no longer holds in the neighboring region. The plasma, in turn, tends to be shielded from the boundary and its potential by this charge-rich layer—called the "plasma sheath." The sheath can support substantial electric fields as the boundary potential is matched to the plasma potential (which we will assume to be zero).

Langmuir and Mott-Smith identified some very special cases (which are applicable to a number of applications of interest; e.g., electrostatic probes in laboratory plasma) which greatly simplify the above phenomena and allows a closed solution to the problem. This simplified treatment applies if:

(1) The distribution of the collected component of the plasma has a Maxwellian distribution at the sheath boundary, $r = r_s$ (which is generally true for the hotter component in a plasma—usually electrons).
(2) Pressure is sufficiently low that $\lambda >> r_s$ and $\lambda >> r_w$ (where λ is the collision mean-free-path length for any species, r_w is the wire radius, and r_s is the plasma sheath radius).
(3) Density, n, or probe potential, ϕ_w, is sufficient low that $\phi_\rho = \phi(r)$; i.e., collisions are unimportant so that the potential within the sheath has a functional (single-valued) monotonically decreasing behavior (where ϕ_w is the electric potential applied to wire).

Additional simplifying assumptions that were applied to the sheath include:

(1) Total absorption of the particles of the collected species that contact the electrode.
(2) No collisional effects on particle trajectories;
(3) No photo-emission from the probe surface
(4) No ionization of neutrals
(5) No recombination of charged particles to form neutrals

(6) No magnetic field effects

(7) Quasi-static conditions (no bulk drift of the plasma)

The resulting current-voltage characteristic is of the form shown in Fig. 1. Section A is the “ion acceleration” or “ion saturation” region. Here, the bias is sufficiently negative to repel all electrons so that only positive ions are collected. Section B is the “electron retarding” region. In this region, the applied probe bias is still negative wrt the plasma, so that electrons are repelled, but current increases as the potential becomes less negative and increasing numbers of electrons in the Maxwellian velocity distribution are able to overcome the potential barrier and be collected. At V_p, essentially none of the electrons are retarded and the complete Maxwellian has been collected. Section C is the “electron acceleration” or “electron saturation” region. Here, the probe bias is sufficiently positive that all electrons of the Maxwellian distribution are able to reach the probe surface and be collected. Accordingly, current increase in this region is primarily the result of an increase of the sheath thickness produced by the increasing electrical bias on the probe. (Some variation of current will occur as the probe potential becomes more positive and ions are increasingly repelled.)

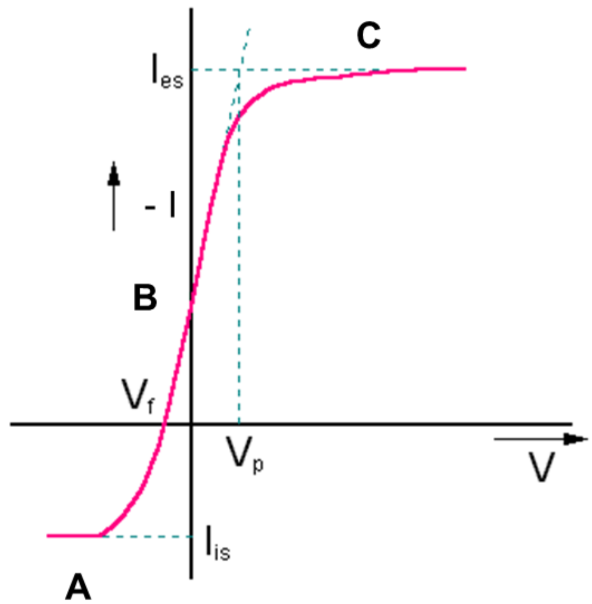

Figure 1. Langmuir Probe i-v Characteristic.

The E-Sail wires are highly biased so that all ions (solar wind protons) are repelled. We are, therefore, only concerned with the current-voltage relation in the electron saturation region. The dependence of sheath expansion, in Section C, on the applied potential is derived below.

III. OML Derivation

If an infinitely long wire is assume, the problem is reduced to two dimensions and the Maxwell distribution for electron velocities can be represented as

$$f(u,v) = \left(\frac{m}{2\pi kT}\right) \exp\left(-\frac{m(u^2+v^2)}{2kT}\right), \tag{1}$$

where u=v_x, v=v_y, k is Boltzmann’s constant, and m and T are the electron mass and temperature, respectively. The Langmuir theory for an infinite cylinder results in an expression for collected current in this region of the form:

$$I = A_w j_{eo} P, \tag{2}$$

where A_w is the area of the wire, j_{eo} is the electron thermal current in the ambient plasma (outside the sheath), and P is an impact parameter. The physical wire area, A_w, is given simply by:

$$A_w = 2\pi r_w L\ , \tag{3}$$

where L is the total length of the wire. The average ambient electron current density produced by a Maxwellian is given by:

$$j_{eo} = \frac{en}{4}\left(\frac{8kT}{\pi m}\right)^{\frac{1}{2}}. \tag{4}$$

The use of an impact parameter, P is a simple means of determining which electrons will impact the wire; i.e., it is assumed that all electrons that enter the sheath and pass within a distance r = P will contact the wire and be collected, while those that pass outside of r = P will escape. The effective collection surface is, therefore, defined by r = P, rather than r = r_w—provided that the above mentioned conditions hold (e.g., no collisions, monotonically decreasing sheath potential, etc.). This is straight-forward for a mono-energetic beam of charged particles, as shown in Fig. 2, but for a thermalized plasma, this requires integrating *uf(u,v)* in Eq. (1), over $0 \le u \le \infty$ and $-v^* \le v \le$

+v*, where v* is the maximum tangential velocity for which a particle will contact the surface of the wire—derived from the conservation of energy and angular momentum as a particle falls through the sheath potential.

This integration leads to an expression for P in cylindrical coordinates, which Huddlestone[5] gives as:

$$P = (\frac{r_s}{r_w} erf\left(Z^{\frac{1}{2}}\right) e^{\Phi}\left[1 - \mathrm{erf}(\Phi + Z)^{\frac{1}{2}}\right], \qquad (5)$$

where Φ is the electric potential normalized by electron thermal energy and z is a function of r_w, r_s, and Φ; i.e.,

$$\Phi = \left(\frac{e\phi_w}{kT}\right), \quad \text{and} \quad z = \left(\frac{{r_s}^2}{{r_s}^2 - {r_w}^2}\right)\Phi,$$

where ϕ_w is the electric potential applied to the wire. P is, therefore, a function of ϕ_w, and allows us to determine how the effective collection area will increase as the applied voltage is increased.

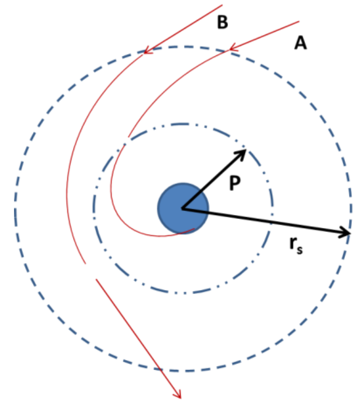

Figure 2. OML assumed electron trajectories.

IV. Application to E-Sail Technology

For a *thick sheath* ($r_s >> r_w$), Z is small compared to Φ, and $\mathrm{erf}(\Phi) \approx (1/\sqrt{\pi})\Phi$. Equation (5) then reduces to:

$$P \cong \frac{2}{\sqrt{\pi}}(\Phi + 1)^{1/2} \approx \frac{2}{\sqrt{\pi}}\Phi^{1/2}, \qquad (6)$$

where the last approximation assumes a high applied electric potential (Φ >> 1). The collected current, from Eqn. (2) and (6), is then, given by:

$$I \cong A_W\, j_{eo}\frac{2}{\sqrt{\pi}}(\Phi)^{\frac{1}{2}}$$

or, inserting the expression for j_{eo} from Eqn. (4), we have:

$$I = A_W\left[\frac{en}{4}\left(\frac{8\mathrm{kT}}{\pi\mathrm{m}}\right)^{\frac{1}{2}}\right]\frac{2}{\sqrt{\pi}}\left(\frac{e\phi_w}{kT}\right)^{\frac{1}{2}}, \qquad (7)$$

Equation (7) simplifies to:

$$I = \frac{en}{\pi}A_W\left(\frac{2e\phi_w}{\mathrm{m}}\right)^{\frac{1}{2}}. \qquad (8)$$

Equation (8) is precisely the expression used by Pekka Janhunen.[1] The attached Excel file contains electron current collection for several cases using equation (8). However, it cannot be too strongly emphasized that, because Pekka used this expression derived from of the LMS formulation, his results (and ours) are subject to all of the limitations inherent to the LMS model.

V. Effects of the Assumptions

Note that, in the Solar Wind, the conditions $\lambda_D >> r_w$ and $r_s >> P$ can exist (where λ_D is the Debye shielding distance). Therefore, in E-Sail applications, λ_D and r_s can be very large so that an error in determining P can have a large effect on the calculation of the collected electron current.

A rigorous treatment of the assumptions and the resulting impact on E-Sail design will require deriving a current collection model in which the assumptions and approximations that impact the validity of the calculations under conditions appropriate for E-Sail operations are removed. It is extremely doubtful that a closed solution can be obtained for such a case and, therefore, obtaining an accurate and trustworthy model will require a substantial investment into the development of (or use of an existing) particle-in-cell numerical model. This is clearly beyond this short discussion. We will, however, point out *some* of the potential issues—which should not be considered an exhaustive list.

(1) The LMS treatment assumes no collisional effects. In essence, this is a requirement that the collected particles (electrons) undergo free molecular flow so that their trajectories through the sheath region are determined totally by their initial velocity (at $r = r_s$) and the sheath electric field.

While the ambient Solar Wind can be considered collisionless on the scale of the sheath radius, this may not be true within the sheath. Because the sheath is large, the sheath field will have the effect of focusing the electron flux crossing a large area into a small volume, thereby raising the density. While this does not appear to have been studied for electron collection, it has been observed in several experimental investigations of the behavior of ions in attractive sheath fields around cylindrical and spherical bodies—and the focused ion fluxes are observed to interact.[5] Electron fluxes would be expected to exhibit a similar behavior.

(2) The LMS treatment also assumes that particles are either collected or lost. However, a third possibility exists: The incoming electrons may go into orbit around the wire and become trapped.

This effect has, again, been observed for the case of ions.[6] Trapping will greatly extend the presence of electrons in the sheath region, thereby contributing to the build-up of a negative space charge. A space charge will modify the electric field in the sheath, which could deflect incoming electrons and/or create a non-monotonic potential distribution—which would violate another of the conditions for application of the LMS model.

(3) The Solar Wind has a very large drift velocity (from which we extract momentum to produce a propulsive force). However, the LMS model assumes quasi-static conditions (plasma drift motion must be small wrt the thermal motion of the plasma constituents), and this is not the case.

The closed solutions for current collection from plasma generally make the quasi-static assumption. On the surface, it appears reasonable since the electron thermal velocity is far greater than either the proton thermal velocity or the bulk drift of most space plasma, including the Solar Wind. However, during the TSS missions, it was discovered that the quasi-static Parker-Murphy model, which had been used to describe the current-voltage characteristic of satelllites for some 30 years, introduced significant errors.

Although still not fully understood, it appears that a relative drift between the plasma and collector can have at least two important effects: (1) it creates a wake region behind the body that can greatly affect current collection and is not accounted for by a static model, and (2) it moves the body continually into fresh flux tubes that are full of ambient plasma (a static model generally assumes that the flux tube, in which the body is placed, will become depleted).

There are a number of other issues (e.g., the existence of a magnetic field imbedded in the Solar Wind) but the above should be sufficient warning against relying heavily on the LMS model to determine E-sail engineering design parameters until, and if, it is found be insensitive to the violations of its underlying assumptions created by condition in the Solar Wind.

VI. Simulation Experiments

The subject experiments were carried out in the Marshall Space Flight Center Space Plasma Chamber.[7] Details of the experiments are available in Stone.[8] The smallest bodies were on the order of a Debye length in radius and had spherical and short cylindrical geometries. The general characteristics of the interaction observed and the measurement technique are shown schematically in Fig. 3. Here a spherical body is charged a few volts negative (comparable to the floating potential in space) and as a result, the streaming ions are deflected into the wake. The width of the region in which the ions were deflected, the "ion deflection" sheath, was determined by measuring the widest point at which the ion velocity deviated from the bulk flow direction (this required vector measurements of ion flux[9]) at a point downstream from the body and extrapolating this back to a plane located at the axial position of the center of the body and oriented normal to the flow direction.

The radial thickness of the sheath normalized by the Debye length in the plasma stream is shown in Fig. 4 for a range of the dimensionless ratio, normalized potential divided by the ion acoustic Mach number. Presenting the data in terms of dimensionless parameters allows the results to be applied more generally. However, in order to use this sheath characterization in the E-sail problem, two primary assumptions are required:

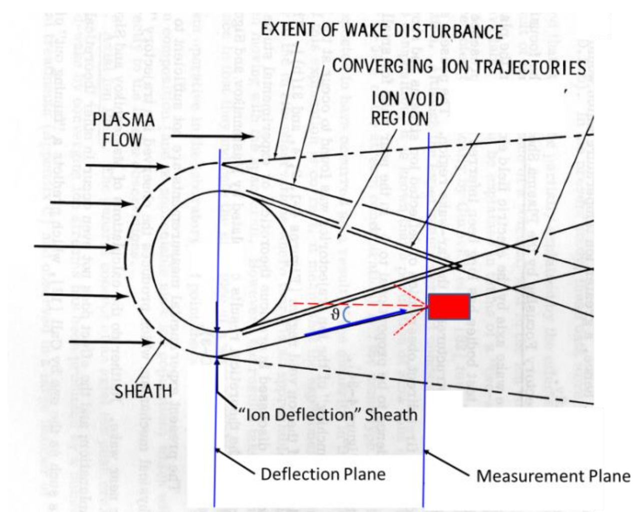

Figure 3. Regions of Disturbed Plasma Flow and Measurement Process.

(1) A positive (repulsive) potential under the same circumstances would deflect the ions over the same amount and over the same volume—just in the opposite direction. In other words, the sheath thickness for detectable ion attraction around a body biased at – ϕ is the same as that for detectable ion repulsion around a body biased at + ϕ.

(2) The spherical geometry should result in a somewhat thinner sheath than would be expected for cylindrical geometry so that the results here should be somewhat conservative—under predicting the size of the sheath and, therefore, the region of proton deflection (under predicting thrust levels) and electron attraction (under predicting current collection).

VII. Experimental Results

Subject to the above caveats, the experimental data can be used to estimate the E-sail thrust. In effect, the OML equation is replaced by the empirically derived relation shown in Fig. 4. Inserting the definitions for Debye length, ion acoustic Mach number, $S = (m_p v^2/2kT_e)^{½}$, and normalized potential, Φ, gives:

$$r_s = r(\phi_w, \vartheta = 90) = 3.0\,(\phi_w)^{\frac{1}{2}}\,, \qquad (9)$$

where r_s is the value of r at which no detectable ion deflection occurs, as shown schematically in Fig. 5. Since the electric field is assumed to be cylindrically symmetric, only the radial component of ion velocity will be acted on. Therefore, at any point of deflection in the sheath located along a radius at an angle ϑ_α, the value of the potential will be given by:

$$\phi(r_\alpha, \vartheta_\alpha) = \frac{1}{2}\left(m_p {v_o}^2\right)cos^2\vartheta_\alpha, \qquad (10)$$

where $(r_\alpha, \vartheta_\alpha)$ such that $r_w < r_\alpha \leq r_s$ and $0^o \leq \vartheta_\alpha \leq 90^o$ defines any position coordinated within the sheath. The electric potential for spherical coordinates between $r = r_w$ and $r = r_s$ is given by:

$$\phi(r) = \phi_w\ \frac{ln(r_s/r)}{ln(r_s/r_w)} \qquad (11)$$

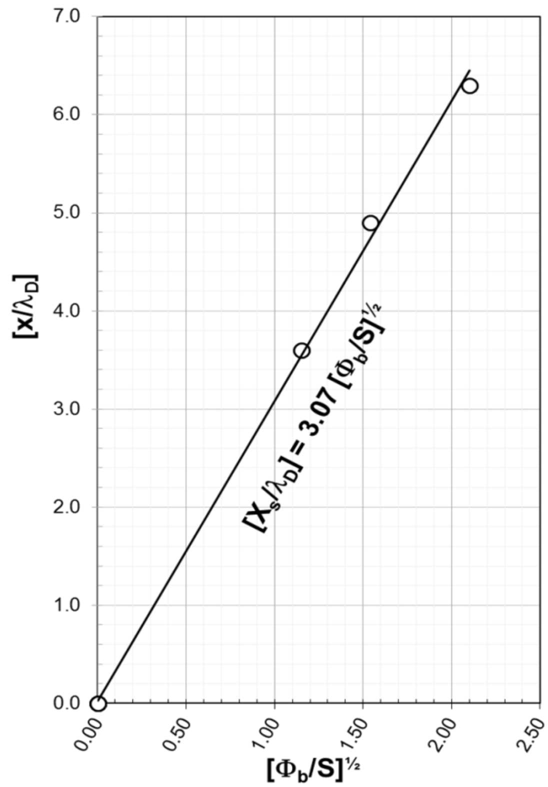

Figure 4. Effective sheath thickness for observable ion deflection.

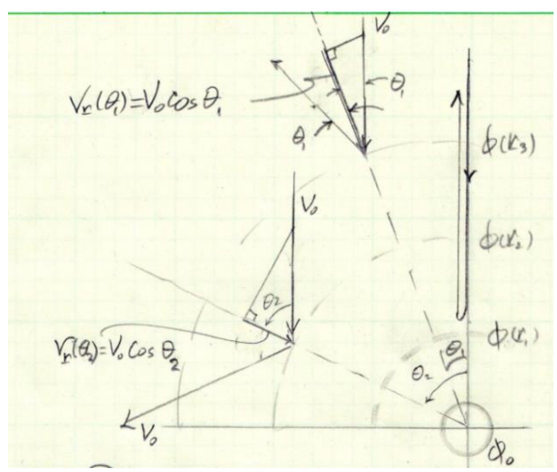

Figure 5. Specular Proton Deflection Calculation.

If we set $r = r_\alpha$* (the point along a radius at an angle ϑ_α at which an incoming ion with velocity v_o is deflected) then the potential defined by Eq. 11 must be equal to the potential required for deflection, given by Eq. 10. Equating Eq. 10 and Eq. 11 and using Eq. 9 to solve for r_s, it is possible to define an envelope of deflection points, r_α*, for any given potential applied to the wire, ϕ_w. Examples for several applied potentials are shown in Fig, 6 where the plasma conditions are typical solar wind values at 1 AU.

From Fig. 5, the force exerted by solar wind protons flux per unit area can be calculated from the loss of momentum $f = n_o v_o(M_{in} – M_{out})$, where $M_{in} = (m_p v_o)$ and $M_{out} = m_p v_o \int_0^{\pi/2} \cos(2\vartheta)\, d\vartheta = 0$. Then $f = n_o\, m_p {v_o}^2$. The propulsive force per meter of wire is then $F = 2r_s(\phi_b)f$, where r_s is determined from Eq. 9. For typical solar wind values at 1 AU (i.e., $n_o = 7x10^6/m^2$, $T=1.5x10^5\ {}^oK$, v_o=400 km/s) the force generated per meter of wire at =6 kV is approximately 0.87 μN/m, whereas values using the OML model are in the range of 0.7 μN/m.

VIII. Conclusion

The discrepancy between the calculated thrust values based on the OML model and the values derived from experimental data obtained in a drifting collisionless laboratory plasma (the experimental based results are ~25 percent higher) are in reasonable agreement for the present level of precision. It should be noted, however, that the experiments used a body geometry and plasma characteristics that are not optimized for an E-sail in the solar wind. The effect(s) of these limitations are not easily predicted, but the electric field might be expected to extend further from a long cylinder than from a body with a square cross section, such as the one used in the experiments discussed herein.

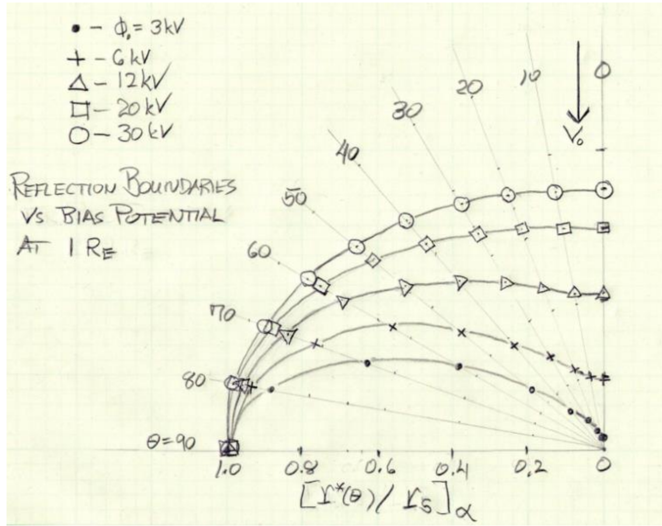

Figure 6. Reflection Boundaries Within Sheath for Varous Proton Energies.

The OML calculation for electron collection is, however, more suspect. The effective collection area is only 24 times larger than that of the physical wire, whereas, the area of the ion deflection sheath (the point at which ion motion is affected) is some $2x10^5$ times greater—and the effective electron sheath can be expected to be even larger. Clearly, most of the electrons entering the sheath will not contact the wire and be collected. However, an impact parameter of only 24 times the wire radius for an applied bias of + 6kV seems small. Recall that the OML model assumes, among other things, no interaction between particles in the sheath, no trapped particles, and a nonotonically decreasing sheath field. These assumptions may not hold when the electrons are first accelerated at a radius that is more than $2x10^5$ times greater than the wire radius. Again, effects not consistent with the OML assumptions have been observed in the behavior of ions accelerated into the sheath; e.g., focusing of ions onto the rear surface of a body when the bias potential exceeded the kinetic drift energy of the streaming ions.[6]

At this point, both approaches (OML and experimental) require assumptions that may affect the derived values of thrust and collected electron current. This suggests that a numerical model that includes all of the governing physics (including plasma drift, collisions, and trapping) be used to develop engineering parameters for a detailed E-sail design. Such a model will be, of necessity, complex and it is, therefore, recommended that new laboratory experiments be carried out that use an appropriate geometry and plasma conditions in order to provide a bench-mark check on the numerical model.

Either answer (whether the OML sheath is approximately correct, or whether the sheth is considerably larger) will have advantages and disadvantages. Neither appears to be a major problem for E-sail technology. However, it is important that the correct engineering design values be reliably established because the answer may affect E-sail accommodation requirements and, therefore, the appropriate class of spacecraft systems. Greater thrust capacity (an advantage for large spacecraft) may also be accompanied by greater electron current collecteion and, therefore, a higher operational power requirement (a disadvantage for small Cubesat type mission).

Acknowledgments

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